An inequality for convex sets.

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M. Henk and myself in [1] proved the following interesting inequality for plane convex sets

$$V < 2Dr$$

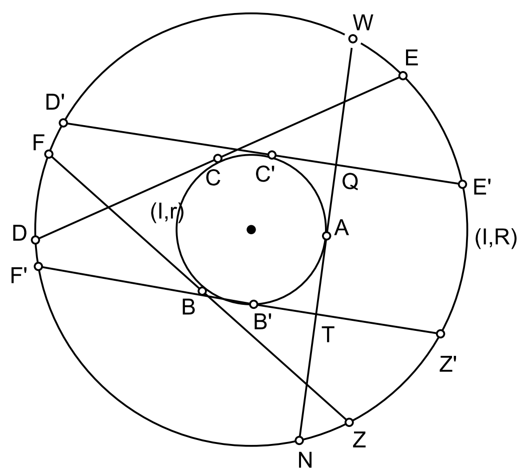
where V is the area, D the diameter and r the inradius of a convex set F. The following proof sems to be simpler. We need the very importand lemma, below.

Lemma
For the plane convex set F holds:

$$D \geq R + r$$

where R the circumradius.
Proof
We consider the circles (I, r) and $(I'D - r)$. We wlll prove that every point $A \in (I, D - r)$ is not an interior point of F. We take at the point A the tangent l_1 to $(I, D-r)$.Let l_2 the parallel tol_1 tangent to (I, r). We see that the distance between l_1 and l_2 is D.
Suppose now n_1 and n_2 are the parallel to l_1, l_2 support lines of F. Let that n_2 is not included in the strip (l_1, l_2). So n_1 must be in the strip (l_1, l_2) and the proof finished.
Therefore $D \geq R+r$. The equality for convex sets with constant width. The same proof for E^n.
Now to the **proof**.
Let (I, r) the inscribed circle in F and $A, B, C \in F \cap (I, r)$ so that arcAB, and arc$AC \geq \frac{1}{2}\pi$(that is possible). We consider the circle $(I, D - r)$. FRom the lemma, we see that every point M of the circle $(I, D - r)$ is not an interior point of F. Let now NW,DE,FZ the tangent lines of (I, r) at the points

A,B,C. The convex set F is included by the lines NW,DE,FZ and the circle $I, D-r)$. So one easy calculation shows that the area of F is max. if F=$QD'F'T$, see fig.



where arcAB'=arcAC'=$\frac{\pi}{2}$.
Therefore $V < area(QD'F'T) < 2Dr$.

References

1. M.Henk,G.Tsintsifas, Some Inequalities for Planar Convex Figures,Elemente der Mathematik 49(1994)
2. T.Bonnesen,W Fencel, Theory of Convex Bodies B Associates.